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Non-Perturbative Green's Functions in Theories with Extended Superconformal Symmetry

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Abstract

The multiplets that occur in four dimensional rigidly supersymmetric theories can be described either by chiral superfields in Minkowski superspace or analytic superfields in harmonic superspace. The superconformal Ward identities for Green's functions of gauge invariant operators of these types are derived. It is shown that there are no chiral superconformal invariants. It is further shown that the Green's functions of analytic operators are severely restricted by the superconformal Ward when analyticity is taken into account.

The most symmetric four dimensional quantum field theory with particles of spin less than or equal to one is the $N = 4$ supersymmetric Yang-Mills theory [1] with four rigid supersymmetries. The action for this theory is uniquely determined by its symmetries once we specify the gauge group. Theories which possess two rigid supersymmetries and have spin less than or equal to one can only be formed from a coupling of $N = 2$ Yang-Mills [2] to $N = 2$ matter [3] These theories are also uniquely determined by their symmetries once we specify their gauge group and the representation of the $N = 2$ matter. In effect, these are the most symmetric rigidly supersymmetric theories, since if we include particles of spin $3/2$ and above then we can only have causal propagation if we also include gravity and so have a theory of local supersymmetry. The special nature of the rigidly extended supersymmetric theories was further confirmed when it was found that the $N = 4$ Yang-Mills theory is finite or superconformally invariant [4] and that the $N = 2$ theories have only a one loop beta function [20,21,5,22]. For a large class of these latter theories it was found [5] that this one loop beta function vanishes so these theories are also superconformally invariant. More recently [6], by exploiting duality, a non-perturbative expression for a certain sector of the effective action of $N = 2$ Yang-Mills with $SU(2)$ gauge group has been deduced. The duality conjecture has also been extended [7] to the simplest finite $N = 2$ theory with $SU(2)$ gauge group.

Since the extended rigidly supersymmetric theories are essentially determined by their symmetries one might hope that one could determine their gauge invariant Green's functions solely by using these symmetries. For the finite theories this symmetry is an extended superconformal supersymmetry. However, it is well known [8] that although (super) conformal symmetries determine the form of two and three point Green's functions they do not, in general, determine the higher point functions. A spectacular exception [9] to this rule is provided by a class of conformally invariant two dimensional models called minimal models. These can be solved as a consequence of the occurrence of null states in the representations of the conformal group which they carry. The elimination of these null states leads to differential equations for the Green's functions which can then be determined.

One might hope that if similar techniques could be used in four dimensional theories then the best candidates would be the most symmetric conformally invariant theories, i.e. theories with extended rigid supersymmetry. In four dimensions the (super) conformal group consists of only the generalisation of those transformations which are globally defined in two dimensions. As is to be expected these can be used to find explicitly the two and three point Green's functions. One indication that one may be able to solve for higher point Green's functions came from the study of the N=2 supersymmetric minimal models in two dimensions. These models contain certain primary fields which correspond to chiral superfields and it was shown [10] that the Green's functions of chiral superfields of the same chirality could be found explicitly using only the globally defined two dimensional superconformal transformations and the chiral constraint which is equivalent to the elimination of one of the null states. This result leads one to hope that one can solve for all the Green's functions which are for chiral superfields of the same chirality in any super-

symmetric theory at a fixed point. Following the earlier work of reference [12], one step in this direction, was taken in reference [11] where it was shown that for a supersymmetric field theory at a fixed point the anomalous dimensions of gauge invariant chiral superfields were determined by their R weight. This result was used [11] to argue for the triviality of the Wess-Zumino model and super Q.E.D.

In this paper, we shall argue that superconfromal symmetry places very strong constriants on Green's functions in any chiral or analytic sector of a superconformal theory. By a chiral Green's function we mean one that involves only all chiral or anti-chiral superfields. Such sectors can found in N=1 and N=2 rigidly supersymmetric theories. In the former they involve the $N = 1$ Wess-Zumino multiplet and the $N = 1$ Yang-Mills field strength, while in the latter they involve the $N = 2$ Yang-Mills field strength. Analytic Green's functions are similarly defined with analytic rather than chiral superfields. Such superfields occur in N=2 and N=4 rigidly supersymmetric theories and are constructed using harmonic superspace [13] as will be explained below. Specifically, analytic superfields can be used to describe the $N = 2$ hypermultiplet [13] and the $N = 4$ Yang-Mills field strength multiplet [14]. An important feature of analytic superfields is that their conjugates are of the same analytic type in contrast to the chiral case. Hence, all multiplets of rigidly supersymmetric theories can be described by chiral or analytic superfields. We shall derive the superconformal Ward identities for correlators involving such superfields and show that there are no chiral invariants and that the form of the four point analytic superconformal invariants is severely restricted.

The chiral and analytic constraints are an important feature of superspace formulations of supersymmetric theories. We recall that a chiral superfield satisfies the constraint $D^i_{\dot\alpha}\phi = 0,\ \dot\alpha = 1, 2,\ \ i = 1, \ldots, N$ where ϕ is the superfield in question, N the number of supersymmetries and $D^i_\alpha \equiv \frac{\partial}{\partial\theta^{\alpha i}} - \frac{i}{2}\theta^{\dot\alpha}_i \frac{\partial}{\partial x^{\alpha\dot\alpha}}$, $D^i_{\dot\alpha} \equiv -\frac{\partial}{\partial\theta^{\dot\alpha}_i} + \frac{i}{2}\theta^{\ \alpha i}\frac{\partial}{\partial x^{\alpha\dot\alpha}}$ are the Minkowski superspace covariant derivatives. The chiral superfields which occur in rigidly supersymmetric theories are the $N = 1$ Wess-Zumino multiplet φ, the $N = 1$ Yang-Mills field strength superfield W_α and the $N = 2$ Yang-Mills field strength superfield W. In fact, these field strengths obey chirality constraints with gauge covariant superspace derivatives, but gauge invariant products of these field strengths obey ordinary chiral constraints. The above superfields are defined on Minkowski superspace which has coordinates $\{x^{\alpha\dot\alpha},\ \theta^{\alpha i},\ \theta^{\dot\alpha}_i\}$, $i = 1, \ldots N$. As a result of the chiral constraint a chiral superfield ϕ depends only on $s^{\alpha\dot\alpha} = x^{\alpha\dot\alpha} - \frac{i}{2}\theta^{\alpha j}\theta^{\dot\alpha}_j$ and $\theta^{\alpha i}$ and can be regarded as a function on chiral superspace. While Minkowski superspace is the coset space of the super Poincaré group divided by the Lorentz group, chiral superspace can also be interpreted as a coset space which is obtained from the complexified super Poincaré group by dividing by a suitable sub-supergroup which is generated by the Lorentz group and a chiral supersymmetry. Chiral constraints on superfields have important consequences for quantum corrections due to the well known non-renormalization theorem [15]. This states that all corrections to the effective action arise as integrals over the whole of superspace and not over a chiral subspace. In fact, if massless particles are present, the chiral superpotential generically receives quantum

corrections [16] (so called non-holomorphicity), the first example of which was found in reference [10].

The other multiplets of rigid supersymmetry , namely $N=2$ matter and N=4 Yang-Mills are also described by constrained superfields. The $N=2$ matter multiplet (the hypermultiplet) was originally described [3] by a scalar superfield $\phi_i,\ i=1,2$ which transforms as a complex doublet of $SU(2)$ and satisfies the superspace constraints

$$D_{\alpha(i}\phi_{j)} = 0 = D_{\dot{\alpha}(i}\phi_{j)} \tag{1}$$

where the internal indices are raised and lowered with the ϵ tensor, for example, $D_{\dot{\alpha}i} = D^{j}_{\dot{\alpha}}\epsilon_{ji}$. The $N=4$ Yang-Mills field strength superfield is a Lorentz scalar superfield W_{ij} which transforms under the six dimensional representation of $SU(4)$, and as such is anti-symmetric, $W_{ij} = -W_{ji}$ and self-dual $W_{ij} = \frac{1}{2}\epsilon_{ijkl}\bar{W}^{kl}$. It satisfies the constraint

$$D_{\alpha i}W_{jk} = D_{\alpha[i}W_{jk]}. \tag{2}$$

In contrast to the chiral constraints, both the above superfields are on-shell and there appear to be no associated analogues of chiral superspace. Using harmonic superspace [13,14], we now discuss the alternative formulations of these multiplets which overcome this drawback.

Harmonic superspace extends the usual Minkowski superspace by an internal coset space. The $N=2$ harmonic superspace is $\hat{M}_2 = M_2 \times S^2$ where M_N denotes N-extended Minkowski superspace. The additional space S^2 can best be thought of as $\frac{SU(2)}{U(1)}$*. For $N=4$, the harmonic space of interest to us here is $\hat{M}_4 = M_4 \times Gr_2(4)$ where $Gr_2(4)$ is the Grassmannian of two planes in $\mathbf{C}^4$. It can be described as the coset space $\frac{SU(4)}{S(U(2)\times U(2))}$. One could equally well consider $\hat{M}_N$ as a coset space of the super Poincaré group with the isotropy group taken to be the product of the Lorentz group and one of the above internal isotropy groups. Superfields on $\hat{M}_N$ are taken to carry induced representations of $G = SU(N)$ with the isotropy group H corresponding to those of the above cosets. Such representations act on fields φ which carry a matrix representation of H denoted by $D(h)$ for any $h \in H$. The fields φ can be regarded as being defined on either $M_N \times G$ or $M_N \times \frac{G}{H}$. In the former case, the field φ is taken to be equivariant with respect to the left action of the isotropy group, meaning it satisfies the relation $\varphi(hg) = D(h)\varphi(g)$, $\forall\ h\ \in\ H$. The induced representation is then defined to act as $U(g_1)\varphi(g) = \varphi'(g) = \phi(gg_1)$, $\forall g, g_1\ \in\ G$. The coset space description can be recovered by choosing a local section of G considered as a left principal H-bundle over $\frac{G}{H}$ or, more plainly, by choosing coset representatives. We denote by y the corresponding coordinates on $\frac{G}{H}$ inherited from those of the group. We define the field ϕ on $\frac{G}{H}$ as $\phi(y) = \varphi(g(y))$. The action of the induced representation on ϕ can be deduced from the above transformation rule to be

$$U(g_1)\phi(y) = D(h(y,g_1))\phi(y'),\ \forall\ g_1\ \in\ G \tag{3}$$

* Throughout this paper all coset spaces are spaces of right cosets.

where

$$yg_1 = h(y, g_1)y'. \tag{4}$$

The infinitesimal form of this equation is given by

$$\delta\phi = V\phi + D(\delta h)\phi, \tag{5}$$

where $V = \delta y^M \frac{\partial}{\partial y^M}$, δy^M are the transformations of the coordinates and the second term is determined from equation (4) by taking $g_1 = 1 + \delta g_1$ and expanding up to first order in variations. Hence, when defined on $\frac{G}{H}$ the fields ϕ have a tensor character corresponding to the above isotropy group transformations. The mapping between coordinate patches on $\frac{G}{H}$ can be realised by a coordinate transformation which is induced by the right action of a group element on $\frac{G}{H}$ and consequently, under this transformation ϕ under goes a H matrix transformation in accord with equation (3).

For $N = 2$ harmonic superspace we denote the group element of $SU(2)$ by $u_I{}^i,\ i = 1, 2$, $I = +, -$. The hypermultiplet can be described by a superfield on φ_+ on $M_2 \times SU(2)$ which satisfies the constraints

$$D_0\varphi_+ = \varphi_+,\ D_+{}^-\varphi_+ = 0, \tag{6}$$

$$D_{\alpha+}\varphi_+ = 0,\ \ D_{\dot\alpha}{}^-\varphi_+ = 0, \tag{7}$$

In the first equation $D_+{}^-, D_-{}^+$ and D_0 are the right-invariant vector fields on $SU(2)$ which are the same as the vector fields induced by the left action of $SU(2)$ acting on itself and D_0 generates the action of the isotropy group, $U(1)$. In the second equation, $D_{\alpha I} = u_I{}^j D_{\alpha j},\ D^I_{\dot\alpha} = u_j{}^I D_{\dot\alpha}^j,\ j = 1, 2;\ I = \{+, -\}$ where $u_i{}^I$ is the inverse of $u_I{}^i$.

The first condition of the equation (6) is equivalent to the equivariant condition discussed above provided φ_+ has $U(1)$ charge 1, while the second condition implies that φ_+ is a holomorphic "function" on $S^2 = \frac{SU(2)}{U(1)}$. The constraints of equation (7) are referred to as Grassmann analyticity constraints [13] and by expanding φ_+ in harmonic variables we find that they imply the relation $\varphi_+ = u_+^i\varphi_i$ where φ_i is the usual $N = 2$ matter field defined on M_2 which obeys the constraints of equation (1). The conjugate superfield $\tilde\varphi_+$ satisfies the identical constraints of equations (6) and (7) which imply that it has the form $\tilde\varphi_+ = u_+^i\bar\varphi_i$.

In the $N = 4$ case, we write elements $u \in SU(4)$ as $u_I{}^i = (u_r{}^i, u_{r\prime}^i),\ r = 1, 2,\ r' = 3, 4,\ i = 1, \ldots, 4$. The $N = 4$ Yang-Mills theory field strength is a superfield W defined on $M_4 \times SU(4)$ subject to the Grassmann analyticity constraints

$$D_{\alpha r}W = 0,\ D_{\dot\alpha}{}^{r'}W = 0, \tag{8}$$

as well as the constraints

$$D_r{}^{s'}W = 0,\ D_0W = 2W. \tag{9}$$

The super field W also satisfies a reality condition which is equivalent to the self-duality of W_{ij}. The above spinor covariant derivatives are defined analogously to the $N = 2$ case, and the right-invariant vector fields on $SU(4)$ are $D_r{}^s$, $D_{r'}{}^{s'}$, D_0 and $D_r{}^{s'}$, $D^s_{r'}$ where the former correspond to the isotropy group and the latter the coset. The superfield W is a singlet under $SU(2) \times SU(2)$, but has $U(1)$ charge 2. The first condition of equation (9) states that W is a holomorphic "function" on $\frac{SU(4)}{S(U(2)\times U(2))}$ and the $U(1)$ constraint implies [14] that $W = \epsilon^{rs} u_r{}^i u_s{}^j W_{ij}$ where W_{ij} is a superfield on Minkowski superspace that satisfies the constraints of equation (2).

In the above, we have worked with superfields defined on the (internal) group, however, for the applications in this paper it will be easier to work with superfields defined directly on the coset spaces as discussed above. We now briefly describe how this works for the hypermultiplet. The group manifold $SU(2)$ can be covered by two patches* U and U' and in U we parameterise group elements by

$$u_I^{\ j} = \frac{1}{\sqrt{(1+y\bar{y})}} \begin{pmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{pmatrix} \begin{pmatrix} 1 & y \\ -\bar{y} & 1 \end{pmatrix}. \tag{10}$$

and in U' by an similar form with y' and ϑ' replacing y and ϑ. On U the right-invariant vector fields on $SU(2)$ are given by

$$D_-{}^+ = (1+y\bar{y})e^{-2i\vartheta}\left(\frac{\partial}{\partial y} + \frac{i}{2}\frac{\bar{y}}{(1+y\bar{y})}\frac{\partial}{\partial \vartheta}\right), \; D_0 = -i\frac{\partial}{\partial \vartheta},$$

$$D_+{}^- = (1+y\bar{y})e^{2i\vartheta}\left(\frac{\partial}{\partial \bar{y}} - \frac{i}{2}\frac{y}{(1+y\bar{y})}\frac{\partial}{\partial \vartheta}\right). \tag{11}$$

On the coset $\frac{SU(2)}{U(1)}$, we choose our coset representatives to be of the above form with $\vartheta = 0$ and $\vartheta' = 0$ in the corresponding two patches, also denoted U and U', on $SU(2)/U(1)$. The transformation between the two patches is given by $y \to y' = \frac{1}{y}$ and it is a straight forward exercise to compute, in term of the parameterisation of equation (10), the group transformation which takes the coset representatives in each patch into each other. One finds that it involves an isotropy rotation for which $e^{2i\vartheta} = \frac{y}{\bar{y}}$ and hence the induced representation is related on the two patches by

$$\phi'(y', \bar{y}') = \left(\frac{y}{\bar{y}}\right)^{\frac{q}{2}} \phi(y, \bar{y}) \tag{12}$$

if ϕ has $U(1)$ charge q, i.e. $\phi \to e^{iq\vartheta}\phi$.

In this coset formulation, the constraints of equations (6) and (7) for φ_+ have a simple interpretation. The D_0 constraint tells us that ϕ_+ has $U(1)$ charge 1. On making the

* The patches correspond to the patches of S^2; strictly one should introduce two patches for the $U(1)$ fibres as well.

change $\phi_+ \rightarrow \frac{1}{\sqrt{(1+y\bar{y})}}\phi_+$ we finds that the last condition of equation (6) states that ϕ_+ is independent of $\bar{y}$. Consequently, ϕ_+ depends on y alone in the patch U and y' in the patch U'. The field ϕ_+ is assumed to be well defined on all of $\frac{SU(2)}{U(1)}$. It is well defined in the patches U and U' if ϕ_+ is polynomial in y around $y = 0$ and polynomial in y' as $y' \rightarrow 0$ respectively. In order for ϕ_+ be globally defined, we must ensure the matching of these two conditions as we change from U to U'. Using equation (12) we find that ϕ_+ can only be of the form $\phi_+ = \phi_1 + y\phi_2$ where ϕ_1 and ϕ_2 are independent of y and are the components of the hypermultiplet in it's original Minkowski space formulation formulation. Thus we recover the above relation between the two formulations, namely $\phi_+ = {u_+}^i\phi_i$.

The spinor constraints of equation (7) imply that ϕ_+ and $\tilde{\phi}_+$ depend only on the variables

$$\lambda^\alpha \equiv \theta^{\alpha 2} - y\theta^{\alpha 1},\ \pi^{\dot\alpha} \equiv \theta_1^{\dot\alpha} + y\theta_2^{\dot\alpha},$$
$$s^{\alpha\dot\alpha} \equiv x^{\alpha\dot\alpha} + \frac{i}{2}\theta_1^\alpha(\theta_1^{\dot\alpha} + y\theta_2^{\dot\alpha}) - \frac{i}{2}(\theta_2^\alpha - y\theta_1^\alpha)\theta_2^{\dot\alpha} \quad \text{and} \quad y\ . \tag{13}$$

Hence, by working on the coset, we can solve all the constraints of equations (6) and (7) and work with fields that only depend on the above coordinates and are polynomials in y of degree at most one. Such superfields can be regarded as functions on a new superspace, called analytic superspace [13] which we now discuss.

Harmonic superspaces, like ordinary superspace can be regarded as coset spaces of the super Poincaré groups. They can also be viewed as coset spaces of superconformal groups [17]. In this paper, we will be concerned with superconformal theories and so we will adopt this latter point of view. In particular, the theories of interest to us involve chiral and analytic superfields and these are best described using chiral and analytic superspaces. However, these superspaces are coset spaces of the complexified superconformal group which is $SL(4|N;C)$ where N is the number of supersymmetries. The relevant isotropy groups are of parabolic type, meaning that, if we regard an element of $SL(4|N;C)$ as made up of four blocks in the obvious way, each block should itself be of block lower triangular form in order for the element to belong to the isotropy group [17]. For example $N = 2$ analytic superspace has an isotropy group consisting of elements of $SL(4|2;C)$ of the form

$$\begin{pmatrix} \times & \times & & & \times & \\ \times & \times & & & \times & \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & & & \times & \\ \times & \times & \times & \times & \times & \times \end{pmatrix}, \tag{14}$$

where $\times$ represents a non-zero entry. The coset representative for this superspace can be chosen to be of the form

$$\begin{pmatrix} I & -is & 0 & -i\lambda \\ 0 & I & 0 & 0 \\ 0 & -i\pi & 1 & y \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{15}$$

where we have the index structure $s^{\alpha\dot\alpha}$, λ^{α} and $\pi^{\dot\alpha}$.

The $N=4$ analytic superspace which relevant for $N=4$ Yang-Mills has an isotropy group consisting of elements of $SL(4|4;C)$ of the form

$$\begin{pmatrix} \times & \times & & & \times & \times & & \\ \times & \times & & & \times & \times & & \\ \times & \times & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times & \times & \times \\ \times & \times & & & \times & \times & & \\ \times & \times & & & \times & \times & & \\ \times & \times & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times & \times & \times \end{pmatrix}, \tag{16}$$

In fact, the superconformal group of $N=4$ is not $SL(4|4;C)$, but rather the simple supergroup obtained from it by factoring out the subgroup consisting of elements proportional to the identity matrix. The coset representative for this analytic superspace can be chosen to be of the form

$$\begin{pmatrix} I & -is & 0 & -i\lambda \\ 0 & I & 0 & 0 \\ 0 & -i\pi & I & y \\ 0 & 0 & 0 & I \end{pmatrix} \tag{17}$$

where y is a two by two matrix whose index structure will be denoted by $y^{aa'}$. Similarly, we have $\lambda^{\alpha a'}$ and $\pi^{a\dot\alpha}$. We note that the bottom right hand corner of the isotropy group elements is the same as the top left corner. The above spaces have compact bodies whereas we wish eventually to consider non-compact Minkowski space. However, there is a correspondence between analytic superspace and Minkowski superspace via harmonic superspace which allows one to restrict to the usual non-compact super Minkowski space, although the internal spaces remain compact [17]. This step is analogous to removing the point at infinity on the Riemann sphere.

We now consider induced representations on the above coset spaces. The fields which carry these representations depend only on s, λ, π and y, and are holomorphic. The $N=2$ matter and $N=4$ Yang-Mills multiplets correspond to particular representations of this type and in this framework are not subject to any constraints beyond the requirement of holomorphicity, meaning they are non-singular. This makes contact with the discussion of these multiplets given above where they were defined on the real coset spaces $\hat{M}_N,\ N=2,4$ subject to the constraints of equations (6) and (7) and equations (8) and (9) respectively. As we have illustrated for the hypermutiplet we can solve the constraints and so recover the analytic coset space with the coordinates of equation (13). For the purposes of this paper it will be simpler to adopt the analytic superspace viewpoint from the outset.

Given a superconformally invariant supersymmetric theory, its Green's functions will obey Ward identities corresponding to the superconformal transformations. Given such a n point

Green's function G for superfields $\phi_p, p = 1, \ldots, n$ of weight q_p, then the Ward identities are of the form

$$\sum_p \{V_M^p + \frac{q_p}{N}\Delta_M^p\}G = 0. \tag{18}$$

where, using equation (5) we identify $\frac{q}{N}\Delta_M\phi \equiv D(\delta h_{M1})\phi$. The subscript M labels the particular conformal transformation. For chiral and analytic superfields these Ward identities take a particularly simple form when expressed in terms of the coordinates of the corresponding chiral and analytic superspaces.

We begin by considering the chiral case first and we shall argue that at a fixed point of any supersymmetric theory the Green's function of gauge invariant chiral operators of the same chirality are determined by superconformal invariance. As explained above, a chiral superfield ϕ is only a function of $s^{\alpha\dot{\alpha}}$ and $\theta^{\alpha i}$. A Green's function G of n chiral superfields will be a function on n copies of chiral superspace, a space which has coordinates $s_p^{\alpha\dot{\alpha}}$, $\theta_p^{\alpha i}$, $p = 1, \ldots, n$. The superconformal Ward-identities for translations, dilations and special conformal transformation are

$$\sum\{\partial_{\alpha\dot{\alpha}}\}G = 0, \tag{19.1}$$

$$\sum\{s^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \frac{1}{2}\theta^{\alpha j}\partial_{\alpha j} + q\frac{(4-N)}{N}\}G = 0, \tag{19.2}$$

$$\sum\{s^{\alpha\dot{\beta}}C_{\dot{\beta}\beta}s^{\beta\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + s^{\alpha\dot{\beta}}C_{\dot{\beta}\beta}\theta^{\beta j}\partial_{\alpha j} + q\frac{(4-N)}{N}s^{\beta\dot{\beta}}C_{\dot{\beta}\beta}\}G = 0, \tag{19.3}$$

those for supersymmetry are

$$\sum\{\partial_{\alpha i}\}G = 0, \tag{19.4}$$

$$\sum\{\theta^{\alpha i}\partial_{\alpha\dot{\alpha}}\}G = 0. \tag{19.5}$$

For the internal symmetry, with traceless parameter $E_j{}^i$, we have the corresponding Ward identity

$$\sum\{\theta^{\alpha j}E_j{}^i\partial_{\alpha i}\}G = 0. \tag{19.6}$$

and finally those for S-supersymmetry are given by

$$\sum\{s^{\beta\dot{\alpha}}\partial_{\dot{\beta}i}\}G = 0, \tag{19.7}$$

$$\sum\{s^{\beta\dot{\alpha}}\theta^{\alpha i}\partial_{\alpha\dot{\alpha}} + \theta^{\beta i}\theta^{\alpha j}\partial_{\alpha j} + q\frac{(4-N)}{N}\theta^{\beta j}\}G = 0. \tag{19.8}$$

In the above equations the sum is over p, however, this index is suppressed on the coordinates and we have used the shorthand notation $\partial_{\alpha\dot{\alpha}} = \frac{\partial}{\partial s^{\alpha\dot{\alpha}}}$ and $\partial_{\alpha i} = \frac{\partial}{\partial\theta^{\alpha i}}$. For $N \neq 4$ we also have R symmetry with the corresponding Ward identity

$$\sum\{\theta^{\alpha j}\partial_{\alpha j} - 2q\}G = 0. \tag{20}$$

Translation invariance, encoded in equation (19.1), implies that G is a function of $s^{\alpha\dot{\alpha}}_{pq} = s^{\alpha\dot{\alpha}}_{p} - s^{\alpha\dot{\alpha}}_{q}$, while the first supersymmetry constraint on G implies that it is a function of $\theta^{\alpha i}_{pq} = \theta^{\alpha i}_{p} - \theta^{\alpha i}_{q}$. Let us now consider a superconformal chiral invariant f, which will satisfy all the superconformal Ward identities with no isotropy group transformations. It will be a function of $s^{\alpha\dot{\alpha}}_{pq}$ and $\theta^{\alpha i}_{pq}$. The R symmetry implies that f will satisfy the constraint

$$\sum\{\theta^{\alpha j}\frac{\partial}{\partial\theta^{\alpha j}}\}f = 0. \tag{21}$$

which implies that it is independent of $\theta^{\alpha i}$. The last supersymmetry constraint of equation (19.5) implies that f is independent of $s^{\alpha\dot{\alpha}}_{pq}$ and so is a constant. Hence, we have shown that there are no superconformal chiral invariants except for a constant. Although R symmetry implies that any solution to the superconformal Ward identities for a given chiral Greens functions has the same power of θ, it is not a legitimate step to divide such nilpotent quantities and so one can not conclude that chiral Greens functions can not contain arbitary functions.

We now consider analytic superfields of $N = 2$ matter and $N = 4$ Yang-Mills. The transformations of these fields are found by using the coset construction discussed above. After some calculation, one finds, for the case $N = 4$, that the results for translations T , dilations D and special conformal transformations K are

$$\delta_T\phi = \{\partial_{\alpha\dot{\alpha}}\}\phi, \tag{22.1}$$

$$\delta_D\phi = \{s^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \frac{1}{2}\lambda^{\alpha a'}\partial_{\alpha a'} + \frac{1}{2}\pi^{a\dot{\alpha}}\partial_{a\dot{\alpha}} + \frac{q}{2}\}\phi, \tag{22.2}$$

$$\begin{aligned}\delta_K\phi = \{&s^{\alpha\dot{\beta}}C_{\dot{\beta}\beta}s^{\beta\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + s^{\alpha\dot{\beta}}C_{\dot{\beta}\beta}\lambda^{\beta a'}\partial_{\alpha a'} + \pi^{a\dot{\beta}}C_{\dot{\beta}\beta}s^{\beta\dot{\alpha}}\partial_{a\dot{\alpha}}\\ &-i\pi^{a\dot{\beta}}C_{\dot{\beta}\beta}\lambda^{\beta a'}\partial_{aa'} + \frac{q}{2}s^{\beta\dot{\beta}}C_{\dot{\beta}\beta}\}\phi,\end{aligned} \tag{22.3}$$

those of supersymmetry are

$$\delta_{S_{\alpha a'}}\phi = \{\partial_{\alpha a'}\}\phi, \tag{22.4}$$

$$\delta_{S_{a\dot{\alpha}}}\phi = \{\partial_{a\dot{\alpha}}\}\phi, \tag{22.5}$$

$$\delta_{S^b_\alpha}\phi = \{-y^{ba'}\partial_{\alpha a'} + i\pi^{b\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\}\phi, \tag{22.6}$$

$$\delta_{S^{b'}_{\dot{\alpha}}}\phi = \{y^{ab'}\partial_{a\dot{\alpha}} + i\lambda^{\alpha b'}\partial_{\alpha\dot{\alpha}}\}\phi, \tag{22.7}$$

those of internal transformations are

$$\delta_{I_{aa'}}\phi = \{\partial_{aa'}\}\phi, \tag{22.8}$$

$$\delta_I\phi = \{y^{aa'}\partial_{aa'} + \frac{1}{2}\lambda^{\alpha a'}\partial_{\alpha a'} + \frac{1}{2}\pi^{a\dot{\alpha}}\partial_{a\dot{\alpha}} - \frac{q}{2}\}\phi, \tag{22.9}$$

$$\delta_{I^{b'}b}\phi = \big\{ - y^{ab'}C_{b'b}y^{ba'}\partial_{aa'} - \lambda^{\alpha b'}C_{b'b}y^{ba'}\partial_{\alpha a'} - y^{ab'}C_{b'b}\pi^{b\dot\alpha}\partial_{a\dot\alpha}$$
$$+i\lambda^{\alpha a'}C_{a'b}\pi^{b\dot\alpha}\partial_{\alpha\dot\alpha} + \frac{q}{2}y^{bb'}C_{b'b}\big\}\phi, \tag{22.10}$$

while those of special supersymmetry are

$$\delta_{SS^{\beta}_{a}}\phi = \big\{is^{\beta\dot\alpha}\partial_{a\dot\alpha} + \lambda^{\beta a'}\partial_{aa'}\big\}\phi, \tag{22.11}$$

$$\delta_{SS^{\dot\beta}_{a'}}\phi = \big\{ - is^{\alpha\dot\beta}\partial_{\alpha a'} + \pi^{a\dot\beta}\partial_{aa'}\big\}\phi, \tag{22.12}$$

$$\delta_{SS^{\dot\beta b}}\phi = \big\{ - is^{\alpha\dot\beta}y^{ba'}\partial_{\alpha a'} + \pi^{a\dot\beta}y^{ba'}\partial_{aa'} - s^{\alpha\dot\beta}\pi^{b\dot\alpha}\partial_{\alpha\dot\alpha} + \pi^{a\dot\beta}\pi^{b\dot\alpha}\partial_{a\dot\alpha} + \frac{q}{2}\pi^{b\dot\beta}\big\}\phi, \tag{22.13}$$

$$\delta_{SS^{\beta b'}}\phi = \big\{iy^{ab'}s^{\beta\dot\alpha}\partial_{a\dot\alpha} + y^{ab'}\lambda^{\beta a'}\partial_{aa'} - \lambda^{\alpha b'}s^{\beta\dot\alpha}\partial_{\alpha\dot\alpha} - \lambda^{\alpha b'}\lambda^{\beta a'}\partial_{\alpha a'} - \frac{q}{2}\lambda^{\beta b'}\big\}\phi, \tag{22.14}$$

In the above equations we have used the shorthand notation $\partial_{\alpha\dot\alpha} = \frac{\partial}{\partial s^{\alpha\dot\alpha}}$, $\partial_{\alpha a'} = \frac{\partial}{\partial\lambda^{\alpha a'}}$, $\partial_{a\dot\alpha} = \frac{\partial}{\partial\pi^{a\dot\alpha}}$ and $\partial_{aa'} = \frac{\partial}{\partial y^{aa'}}$. These transformations are for $N = 4$ analytic superfields which have $U(1)$ charge q where the $U(1)$ is the obvious $U(1)$ in the $S(U(2)\times U(2))$ isotropy group. When using the constrained superfields this is equivalent to the constraint $D_0\phi = q\phi$.

We can obtain the superconformal transformations for an $N = 2$ analytic superfield of charge $U(1)$ q (i.e. $D_0\phi = q\phi$) by making the following substitutions in equation (22)

$$\lambda^{\alpha a'} \to \lambda^{\alpha}, \quad \pi^{a\dot\alpha} \to \pi^{\dot\alpha}, \quad y^{aa'} \to y \tag{23}$$

and $\frac{q}{2} \to q$ in equations (22.1), (22.2) and (22.3). We must also add the R transformations which are of the form

$$\delta_R\phi = \big\{\lambda^{\alpha}\partial_{\alpha} - \pi^{a\dot\alpha}\partial_{a\dot\alpha}\big\}\phi \tag{24}$$

since such transformations are not present for $N = 4$.

Observables in rigidly supersymmetric gauge theories divide into two types, non-local observables such as Wilson loops and local gauge invariant operators constructed from products of the elementary fields of the theory. Our aim will be to calculate Green's functions of these latter observables using the superconformal Ward identities of equation (18) and the transformations of equation (22). For $N = 1$ supersymmetric theories these local observables can only be constructed from W_α, ϕ, and their conjugates which are anti-chiral. As we have seen above, the superconformal Ward identities only determine Green's functions in the chiral or anti-chiral sector. For $N = 2$ theories, the basic superfields are the Yang-Mills superfield strength W and its conjugate $\bar W$ and the N=2 matter superfield ϕ_+ and its conjugate $\tilde\phi_+$. A very important point for our analysis is that while the $N = 2$ Yang-Mills superfield W is chiral, its complex conjugate $\bar W$ has constraints of the opposite chirality, while the superfield for N=2 matter and its complex conjugate have the same chiral constraints. As we shall see the superconformal Ward identities only determine

Green's function of operators in a given chiral or analyticity sector. However, since N=2 matter has the same analyticity properties as its conjugate we can hope to restrict all such Green's function involving any gauge invariant $N = 2$ matter, although we can still only determine the Green's functions of the chiral or anti-chiral sectors of gauge invariant products of the $N = 2$ Yang-Mills superfield.

Although the $N = 4$ Yang-Mills multiplet is described by a real single analytic superfield W, it is not the case that all composite multiplets can be obtained as products of W. To see this consider the example of $TrW_{ij}W_{kl}$; this product decomposes into a $\underline{20}$ and a $\underline{1}$ under $SU(4)$, but only the $\underline{20}$ is present in TrW^2 as may easily be checked. In the harmonic formalism, we can define a superfield $W' = \epsilon^{r's'}u_{r'}{}^{i}u_{s'}{}^{j}W_{ij}$ which is anti-analytic. The product $TrWW'$ includes both the $\underline{20}$ and the $\underline{1}$. We can hope to use the superconformal Ward identities to determine all Green's functions of gauge invariant polynomials of the superfield W.

To get a feel for the way the superconformal Ward identities act, we now calculate the two point function, G_{12} in $N = 4$ Maxwell theory. This is a function of $s_p^{\alpha\dot{\alpha}}$, $\lambda_p^{\alpha a'}$, $\pi_p^{a\dot{\alpha}}$ and $y_p^{aa'}$, $p = 1, 2$. Taking the transformation equation to mean also the corresponding Ward identity, equations (22.1), (22.4),(22.5),(22.8) respectively imply that G_{12} is a function of their differences $s^{\alpha\dot{\alpha}} = s_1^{\alpha\dot{\alpha}} - s_2^{\alpha\dot{\alpha}}$, $\lambda^{\alpha a'} = \lambda_1^{\alpha a'} - \lambda_2^{\alpha a'}$, $\pi^{a\dot{\alpha}} = \pi_1^{a\dot{\alpha}} - \pi_2^{a\dot{\alpha}}$ and $y^{aa'} = y_1^{aa'} - y_2^{aa'}$. The Green's function consisting of two W's ,which have $q = 2$ corresponding to the fact that they have dilation weight one, has its form fixed by equation (22.2) and Lorentz invariance to be given by

$$G_{12} \equiv < W(1)W(2) >= \frac{h(y)}{s^2} + \frac{f(y)}{(s^2)^2}\lambda^{\alpha a'}\pi^{a\dot{\alpha}}s_{\alpha\dot{\alpha}}y_{aa'} + \ldots, \tag{25}$$

where $+\ldots$ denote terms of higher order in anti-commuting variables, $s^2 = s^{\alpha\dot{\alpha}}s_{\alpha\dot{\alpha}}$ and h and f are functions of $y^{aa'}$. Equation (22.9) implies that $f \propto y^0$ and $h \propto y^2$ which taken with the other internal identities imply that $h = y^2 \equiv y^{aa'}y_{aa'}$ and f is a constant. The Green's function can now be obtained, for example, by using equation (22.6) and the final result can be written in the form

$$G_{12} = \frac{\hat{y}^2}{s^2}, \tag{26}$$

where $\hat{y}^{aa'} = y^{aa'} - 2i\frac{\lambda^{\alpha a'}\pi^{a\dot{\alpha}}s_{\alpha\dot{\alpha}}}{s^2}$. Since one does not have R invariance in $N = 4$ Yang-Mills theory one can in principle have a λ^2 terms, however, one can verify that no such terms are possible for the two point function.

One can carry out an analogous calculation for the two point function for $N = 2$ matter, the result is of the form of equation (26) with the replacements of equation (23).

We can construct two point and higher point Green's functions in $N = 4$ Yang-Mills by taking product of appropriate powers of the above two point function. These will also

satisfy the superconformal identities of equation (18), since these contain operators that obey the Leibnitz rule. As we are interested in gauge invariant operators we consider TrW^n which obeys the same constraints as W of equations (8) and (9) except that $D_0 TrW^n = 2nTrW^n$. As a result, we may satisfy the superconformal Ward identities by taking

$$< TrW^{n_1}(1)\ldots TrW^{n_N}(N) >= \prod_{i<j} G_{ij}{}^{\frac{1}{(N-2)}(n_i+n_j-\frac{n}{N-1})}, \tag{27}$$

where $n = \sum n_i$.

The above expression is not unique as we can form the invariants such as

$$\hat{u} = \frac{G_{13}G_{24}}{G_{12}G_{34}},\ \hat{v} = \frac{G_{14}G_{23}}{G_{12}G_{34}}, \tag{28}$$

and we can multiply the above expression by an arbitrary function of these and still obtain Green's functions that satisfy the superconformal Ward identities. However, one must take care to ensure that analyticity in $y^{aa'}$ is preserved. Since the highest power of $y^{aa'}$ is encoded by the internal identities of equation (22.9), this condition means that one should have Green's functions which have no poles in y^2. As a result, all possible Green's functions constructed in this way are products of two point functions. The above invariants can be written in the form $\hat{u} = \frac{u}{\hat{t}},\ \hat{v} = \frac{v}{\hat{w}}$ where $u = \frac{s_{12}^2 s_{34}^2}{s_{13}^2 s_{24}^2}$, $v = \frac{s_{12}^2 s_{34}^2}{s_{14}^2 s_{23}^2}$, $\hat{t} = \frac{\hat{y}_{12}^2 \hat{y}_{34}^2}{\hat{y}_3^2 \hat{y}_{24}^2}$, $\hat{w} = \frac{\hat{y}_{12}^2 \hat{y}_{34}^2}{\hat{y}_{14}^2 \hat{y}_{23}^2}$.

Any Green's function in $N = 4$ Yang-Mills theory can be written as the product of two point functions that appear in equation (27) times a superconformal invariant. In particular, the four point Green's fnction can be written as the product of G_{12} and G_{34} to suitable powers times a superconformal invariant depending on four points in analytic superspace. Hence, the first step in determining the four point Green's function is to find what the four point superconformal invariants are. Clearly, any function of the $N = 4$ analytic invariants, $\hat{u},\ \hat{v}$ is an invariant, but we can ask if these the only independent invariants. Invariants are quantities which obey all the superconformal Ward identities of equation (18) with $q = 0$, but they need not be analytic functions of the y_{ij}. An invariant can only be of the form

$$I = f + \lambda_{mn}^{\alpha a'} \pi_{pq}^{a\dot{\alpha}} H_{\alpha\dot{\alpha};aa'}^{mn;pq} + \ldots \tag{29}$$

where we have already used the fact that the invariant must be a function of differences and $mn = 12, 23, 34$. Equations (22.3) and (22.10) imply that f can be regarded as only a function of $\hat{u},\ \hat{v}, t$ and w. The strategy is to find the y and s dependence of those $H^{mn;pq}$'s which involve mn and $pq = 12$ or 34 using the non-linear internal and non-linear special conformal Ward identities respectively. We then solve for the remaining components of $H^{mn;pq}$ using the Q-supersymmetry identities and then finally use S-supersymmetry to show that $H = 0$ and that f does not depend on t and w. This calculation is very lengthy and we only give the briefest outline. To illustrate the calculation we carry it out making the assumption that $H_{\alpha\dot{\alpha};aa'}^{mn;pq}$ can be expressed in terms of $(s_{ij})_{\alpha\dot{\alpha}}$times coeficients that

depend on the values of a, a', m, n, p and q. Using equations (22.3) and (22.10) we find that

$$H^{12:12}_{\alpha\dot\alpha;aa'} = \sum_{pq} \frac{(y_{1p})_{aa'}}{y^2_{1p}} \frac{(s_{1q})_{\alpha\dot\alpha}}{s^2_{1q}} h^{12,12}_{1q;1p},\ H^{12:34}_{\alpha\dot\alpha;aa'} = \frac{(y_{14})_{aa'}}{y^2_{14}} \frac{(s_{14})_{\alpha\dot\alpha}}{s^2_{14}} h^{12,34},$$

$$H^{34:34}_{\alpha\dot\alpha;aa'} = \sum_{pq} \frac{(y_{p4})_{aa'}}{y^2_{p4}} \frac{(s_{q4})_{\alpha\dot\alpha}}{s^2_{q4}} h^{34,34}_{q4;p4},\ H^{34:12}_{\alpha\dot\alpha;aa'} = \frac{(y_{14})_{aa'}}{y^2_{14}} \frac{(s_{14})_{\alpha\dot\alpha}}{s^2_{14}} h^{34,12}. \tag{30}$$

where the h's depend on only $\hat u,\ \hat v,\ t$ and w. We now use equations (22.6) and (22.7) to solve for the remaining components of H. For example, one finds that

$$H^{12;23}_{\alpha\dot\alpha;aa'} = \sum_{p} \frac{(y_{13})_{aa'}}{y^2_{13}} \frac{(s_{1p})_{\alpha\dot\alpha}}{s^2_{1p}} h^{12,12}_{1p;13} + \frac{(y_{14})_{aa'}}{y^2_{14}} \frac{(s_{14})_{\alpha\dot\alpha}}{s^2_{14}} h^{12,34}. \tag{31}$$

Finally, substituting equations (30) and (31) into (22.12) one discovers that $H^{12;12}_{\alpha\dot\alpha;aa'} = 0 = H^{34;12}_{\alpha\dot\alpha;aa'} = \frac{\partial f}{\partial t} = \frac{\partial f}{\partial w}$. Solving for the other components of H from the supersymmetry identities and using equation (22.11) one finds that all the components of H vanish.

This result illustrates the strength of the superconformal Ward identities. However, the above assumption on the form of $H^{mn;pq}_{\alpha\dot\alpha;aa'}$ is not the most general since for fixed values of a, a', m, n, p and q , $H^{mn;pq}_{\alpha\dot\alpha;aa'}$ can take on four values while $(s_{ij})_{\alpha\dot\alpha}$ has only three values $(s_{12})_{\alpha\dot\alpha}$, $(s_{23})_{\alpha\dot\alpha}$ aqnd $(s_{34})_{\alpha\dot\alpha}$. The most general form of $H^{mn;pq}_{\alpha\dot\alpha;aa'}$ would include a term of the form $(s_{34}s_{23}s_{12})_{\alpha\dot\alpha}$. This is equivalent to including the term $\epsilon^{\mu\nu\rho\tau}(s_{12})_\nu(s_{23})_\rho \times (s_{34})_\tau(\sigma_\mu)_{\alpha\dot\alpha}$. The full calculation including this term is too complicated to present in this paper, but one finds that in addition to $\hat u,\ \hat v$ there are two other invariants. Their precise form will be given in reference [18].

Although preliminary calculations suggest otherwise, one could, in principle, have four point invariants which begin with $(\lambda\pi)^2$ or higher powers. However, invariants of this type can only involve functions of $\hat u$ and $\hat v$ as a consequence of the above argument. Since R symmetry is not a symmetry of $N = 4$ Yang-Mills theory it might seem that there could be terms of the form $\lambda^2 + \cdots$ etc. R transformations act only on the spinor coordinates by $\lambda \to a\lambda,\ \pi \to \bar a\pi$, where $|a| = 1$: it commutes with the superconformal transformations and as a result, the superconformally invariant Green's functions can be classified according to their R weights and calculated separately. However, $N = 4$ Yang-Mills theory is invariant under the Z_4 subgroup of R transformations (i.e. $a^4 = 1$) under which the $N = 4$ Yang-Mills field strength transforms as $W \to -W$. This symmetry forbids, for example, the occurrence of $\lambda^2 + \cdots$ terms in the four point invariant.

The function of the invariants appearing in a Green's function is not arbitrary, but must be such that the Green's function is an analytic function of $(y_{ij})_{aa'}$. This places very strong constraints on the form of the function of the invariants and we believe that for four point Green's functions composed of a suitable class of operators analyticity is sufficiently strong

to determine the function up to constants. Details of this result can be found in references [18],[23] and [25].

Using similar techniques one can show very similar results for the $N = 2$ matter. In particular, if we take a restricted form as discussed above for the form of H one finds that the only four point superconformal invariants are $\hat{u}$ and $\hat{v}$. However, if we take the most general form of H then one finds one more independent invariants [18]. Writing the four point Green's function as the product of two point functions times an invariant one can then show that for a suitable class of low dimension operators that the function of the invariants is severly restricted [18,23] and is determined for certain operators [25].

To summarise, we have formulated the superconformal Ward identities in the analytic sectors and shown that they and analyticity place strong constraints on the form that Green's functions can take.

In this paper we have only discussed superconformal theories. However, it should be possible to apply similar techniques to the spontaneously broken theories and perhaps to the pure $N = 2$ Yang-Mills theory where one has anomalous as well as spontaneous breaking of superconformal invariance. These topics are under investigation [24] and it is hoped to make contact with the work of Seiberg and Witten [6,7]. It is possible that the non-perturbative methods outlined in this paper could shed further light on duality, which could be seen as an inevitable consequence of the powerful constraints resulting from the symmetries of the theories.

Note Added Although all the calculations given in the original paper were correct some conjectures and claims for future work were not. In the revised version we have changed these statements so that they are, with the benefit of hindsight, correct and we have referred to where these results can be found in our later work. In fact, it turns out that extremal Green's functions are soluble as a result of analyticity and superconformal Ward identities [25] so that the general thrust of the conjecture in the original paper is correct for some special choices of the charges of the operators. These correlators were discussed from the AdS point of view in [26,27] and a perturbative verification at the one loop level was given in [28]. However, in the original paper, the conjecture was claimed for a larger class of correlation functions. In a recent paper [23] the full details of the restrictions that analyticity and superconformal symmetry place on four-point functions of charge two operators in $N = 2$ have been presented; it is shown that, for this choice of charges, these retrictions do not fully determine the Green's functions.

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